International Journal of Scientific and Innovative Mathematical Research (IJSIMR)
Volume 6, Issue 9, 2018, PP 25-30
ISSN No. (Print) 2347-307X & ISSN No. (Online) 2347-3142
DOI: http://dx.doi.org/10.20431/2347-3142.0609004
www.arcjournals.org

Detecting Habitable Exoplanets During Asteroidal Occultations

Abdul Ahad

Luton, Bedfordshire, United Kingdom

Corresponding Author:** ***Abdul Ahad, *Luton, Bedfordshire, United Kingdom, ahad8307@gmail.com.*

Abstract:*This paper sets out a mathematical model for determining the apparent visual magnitude of an extrasolar planet situated within the habitable zone of its parent star, as seen from Earth, and makes the case that such bodies can be detected with ground based instruments during asteroidal occultations of bright nearby stars, such as Alpha Centauri A/B, Sirius A, Procyon A, Altair, Vega and Fomalhaut. The Alpha Centauri system is focused upon as a target case, where apparent magnitudes of hypothetical exoplanets are projected, possessing either Earth or Jupiter-like albedo (reflectivity) properties, a range of physical sizes and changing phase illuminations as they revolve about their parent stars.*

1. INTRODUCTION

Extrasolar planets circling around other stars can, in some cases, be directly photographed by using any number of techniques which mask out the light of the parent star itself, thus enabling the faint planetary bodies within its vicinity to be imaged. This would prove especially feasible for stellar candidates in the Solar neighborhood, where the angular separation between the star and any orbiting planets would be the greatest. Any success in directly imaging a planet situated within the life-supporting habitable zone would clearly be of even greater significance, for obvious reasons. With this approach the question that inevitably arises is this: at what visual magnitude would such a habitable exoplanet be expected to shine when its light has been successfully isolated from that of its parent star in the manner just described? I had first posted a solution to this in a *Usenet* forum back in the fall of 2004 which formed the basis for this paper[1].

2. BASIS OF MODEL

The brightness ratio, R, between any two sources of light whose apparent visual magnitudes are m_1 and m_2, is given by:

$$R = 10^{0.4|(m_2-m_1)|} \qquad (1)$$

where the index quantity $|(m_2-m_1)|$ denotes the absolute value of the difference in magnitude.

The brightness ratio given by equation (1) applies equally to both emitting sources of light, such as stars, galaxies and quasars, as well as reflecting ones, such as planets, moons, asteroids and comets. Absolute magnitudes for bodies shining by reflected sunlight within our Solar System are sometimes measured based on a standard photometric parameter known as “V(1,0)”.

If the Earth, for example, were hypothetically placed at a standard distance of 1 Astronomical Unit (AU) from both the observer *and* the Sun, and it exhibited a phase of 100% (full disk), our planet would be expected to shine at an apparent visual magnitude of -3.86. This quantity is denoted V(1,0) in the literature and its values for each of the planets within our Solar System are listed in NASA factsheets[2].

The Sun also shines from an average distance of 1 AU away from us and it has an apparent visual magnitude of -26.8. Hence, if both the Earth and the Sun were viewed from a standard distance of 1 AU, then by equation (1) above we note that the Earth would be overpowered by the Sun by a total brightness factor of some 1,499,684,836 to 1 (or ~1.5 billion to 1).

Through similar calculations, we find that the planet Jupiter’s brightness ratio to the Sun, if it were placed at a standard distance of 1 AU from both the observer and the Sun, and it exhibited a phase of 100%, (its V(1,0) is-9.40) would be about 9,000,000 to 1.

Referring again to equation (1) above, let m_1 be the apparent visual magnitude of the brighter source (i.e. the Sun) and let m_2 be the apparent visual magnitude of the fainter source (i.e. the planet). Then, taking logarithms to base 10 on both sides and rearranging, equation (1) takes on the form:

$$m_2 = 2.5 \text{ Log}_{10} R + m_1 \tag{2}$$

Let us suppose the Sun is being viewed from an imaginary remote location in space at the distances of the nearby stars, from where it takes on an utterly starlike appearance and shines at a V-magnitude of m_1. Since we know the brightness ratios, equation (2) enables us to accurately determine the expected visual magnitude, m_2 of either the Earth or Jupiter, if either were placed in the habitable zone around our Sun, as seen from such a remote location.

3. APPARENT MAGNITUDES OF EXOPLANETS ORBITING AROUND OTHER STARS

If the Earth were hypothetically placed in the habitable zone around another star and it were viewed from a remote location out in deep space, then by the default definition of a "habitable zone" its surface would be exposed to the equivalent amount of light and heat flux which it experiences in our own Solar System. In order for water to exist in a liquid form on the surface of any planet, it must receive a similar level of light and heat intensity from its parent star as the Earth does from our Sun, regardless of the star's own intrinsic luminosity or spectral classification. Hence, the brightness ratio between that star and any Earth-like habitable exoplanet will always be constant, irrespective of the candidate star concerned. Similarly, if Jupiter were hypothetically placed in the habitable zone of another star, we would expect an identical brightness ratio to that in our own Solar System. Hence, the model described in this paper can be utilised to predict the apparent visual magnitude of an exoplanet orbiting around *any* star within its habitable zone, as seen from Earth. Conversely, if the apparent magnitude of such a planet were to be estimated from direct visual observations, we can deduce an approximation for its size and mass, based upon it possessing either Earth or Jupiter-like physical and photometric properties.

We note further that the apparent visual brightness of an exoplanet is directly proportional to the apparent visual brightness of its parent star. There are just 7 exceptionally bright stars within 30 light-years from the Sun, namely: Alpha Centauri A/B, Sirius A, Procyon A, Altair, Vega and Fomalhaut which therefore form the most likely candidates for the ground-based detection possibilities of potentially large sized exoplanets discussed in this paper.

4. APPLICATION OF MODEL TO THE ALPHA CENTAURI SYSTEM

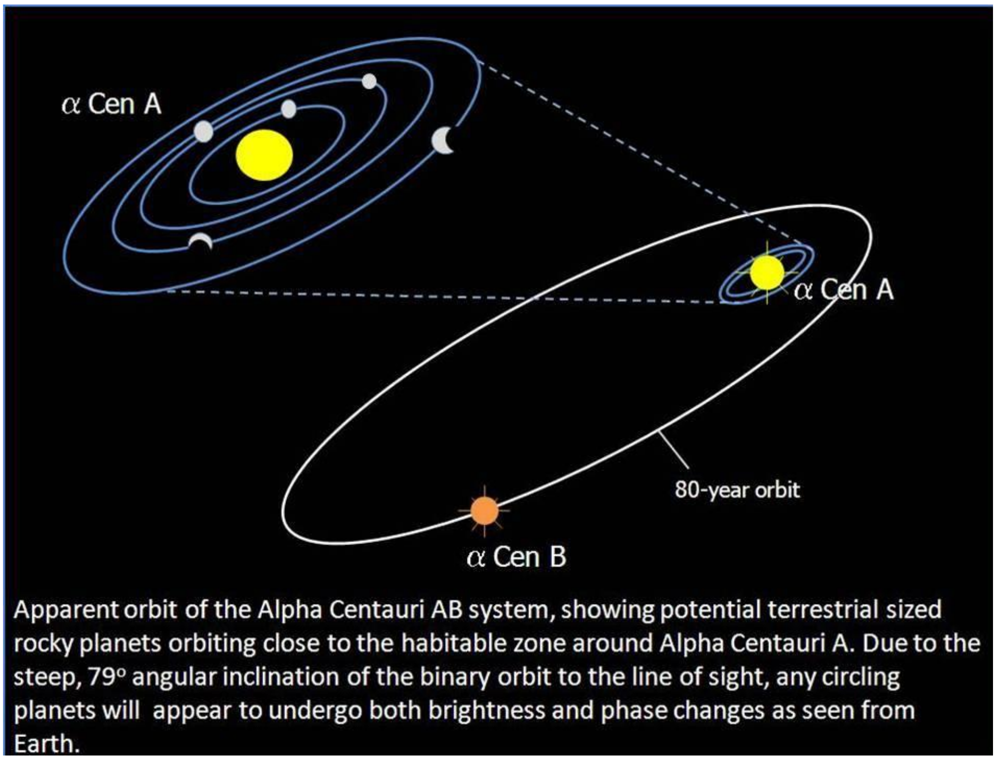
In an imaginary scenario, let us suppose the Earth is placed in the habitable zone around the nearby Sun-like star Alpha Centauri A, which shines at an apparent visual magnitude of -0.01. Then, by equation (2), its apparent magnitude in the neighbourhood of that star as seen from our Solar System would be:

$$m_2 = 2.5 \text{ Log}_{10} (1{,}500{,}000{,}000) + (-0.01) = 22.9$$

This figure is of course based upon the assumption that the disk of the Earth is presented at 100% phase. In actual practice, due to changing orbital orientations, exoplanets would undergo variations in phase, and hence, in brightness. If the Earth, circling Alpha Centauri A, were viewed at only 50% phase, for example, then equation (1) can be used to reduce its brightness by an equivalent factor to yield an expected fainter apparent magnitude of 23.7.

It has been determined from dynamical studies[3] that each of the two principal A and B components of the Alpha Centauri triple star system can comfortably hold up to four terrestrial sized rocky planets in orbit close enough to the habitable zones around each star, without them suffering too much gravitational disruption from the other star, as illustrated in Figure 1.

The habitable zones around Alpha Centauri A and B were determined by Weigert and Holman (1997)[3] to exist at about 1.25 and 0.73 AUs out from each star, respectively. Apparent magnitudes for any prospective planets that might reside in these zones are shown in Tables 1 and 2.



Apparent orbit of the Alpha Centauri AB system, showing potential terrestrial sized rocky planets orbiting close to the habitable zone around Alpha Centauri A. Due to the steep, 79° angular inclination of the binary orbit to the line of sight, any circling planets will appear to undergo both brightness and phase changes as seen from Earth.

Figure1. *The principal components of the Alpha Centauri system, showing imaginary habitable planets orbiting in the orbital plane of the AB system.*

Table1. *Planets Orbiting in the Habitable Zone around Alpha Centauri A*

Planet Diameter	V(1,0)	Brightness Ratio vs Sun	V-mag at 100% Phase	V-mag at 50% Phase
0.5 x Earth	-3.11	2,992,000,000	23.7	24.5
1.0 x Earth	-3.86	1,500,000,000	22.9	23.7
2.0 x Earth	-4.60	759,000,000	22.2	22.9
0.25xJupiter	-7.89	37,000,000	18.9	19.7
0.5 x Jupiter	-8.65	18,000,000	18.1	18.9
1.0 x Jupiter	-9.40	9,000,000	17.4	18.1
2.0 x Jupiter	-10.15	5,000,000	16.6	17.4
3.0 x Jupiter	-10.59	3,000,000	16.2	17.0

Table2. *Planets Orbiting in the Habitable Zone around Alpha Centauri B*

Planet Diameter	V(1,0)	Brightness Ratio vs Sun	V-mag at 100% Phase	V-mag at 50% Phase
0.5 x Earth	-3.11	2,992,000,000	24.9	25.7
1.0 x Earth	-3.86	1,500,000,000	24.2	24.9
2.0 x Earth	-4.60	759,000,000	23.4	24.2
0.25xJupiter	-7.89	37,000,000	20.2	20.9
0.5 x Jupiter	-8.65	18,000,000	19.4	20.2
1.0 x Jupiter	-9.40	9,000,000	18.6	19.4
2.0 x Jupiter	-10.15	5,000,000	18.0	18.6
3.0 x Jupiter	-10.59	3,000,000	17.4	18.2

Viewed from our vantage point here on Earth, a planet circling around Alpha Centauri A at the centre of its habitable zone would have a maximum apparent separation of 0.94arcsecond; one circling around its companion star, Alpha Centauri B, would have a maximum apparent separation of 0.55arcsecond. Assuming circular orbits, these are the maximum planet-star angular separations. Thus in both cases, habitable exoplanets would be within fairly easy resolution range of even amateur-sized instruments, though actually seeing them would be a staggering challenge. The apparent magnitudes listed in Tables 1 and 2 above for potentially larger sized, Jovian-type objects would be within the grasp of large telescopes. It is not unthinkable that a large exoplanet, having a diameter larger than that of our Jupiter, could be attended by a family of terrestrial sized moons and that any one of them could turn out to be a life-bearing world, providing the whole system is orbiting the parent star within the habitable zone. What makes the detection of exoplanets so difficult with ground-based instruments is of course the problem of containing the comparatively vast amount of scattered light coming from the star itself, which drowns out the faint planetary body with an overwhelming amount of noise in the CCD imaging instruments.

5. SOME LIMITATIONS WORTH NOTING

The model formulated in this paper and its results have been projected in visual magnitude terms, based upon the light flux from our own Sun (spectrum G2V). The particular wavelengths at which the planet imaging instruments are operating may be significant, since some extrasolar planets shine more brightly when viewed in the mid- to far infrared part of the electromagnetic spectrum compared to visual wavelengths.

For ground based observations, atmospheric extinction must also be factored into the magnitude values. Atmospheric extinction in stellar magnitudes can of course be minimized by viewing the candidate star (along with any accompanying exoplanets), when such a star is placed at the observer's zenith and when observed from a high altitude location where the air is much thinner, compared to sea level. In such cases, the magnitude values stated above can be taken as they are, without any appreciable reduction due to atmospheric refraction.

6. ASTEROIDAL OCCULTATIONS

A novel way of completely masking out the light of the parent star would be to observe the candidate star during an asteroidal occultation. Exceptionally good imaging opportunities could arise, for instance, when one of these stars is occulted by an extremely faint 16th or 17th magnitude asteroid, when the brightness of the parent star would momentarily dim to the same level as that of the occulting body, allowing the encircling planets to briefly 'peek out' for a rare photo opportunity. Occultation durations typically range from a few tenths of a second all the way up to anywhere near 10 to 15 seconds, allowing similar exposure times that would be greater for asteroids occulting near their stationery points pre and post opposition or slower moving Trans-Neptunian objects. Amateur astronomers currently utilise high-speed super sensitive video cameras, such as the Supercircuits PC164CEX-2, that allow one to image stars as faint as 13th magnitude with a 12 inch telescope at 30 frames (60 fields) per second. Of course exoplanets are much fainter than this, so significantly larger aperture telescopes would be needed.

Occultation of bright stars occur sporadically. The first magnitude star Alpha Leonis (Regulus), for instance, was occulted by magnitude 12.4 asteroid163 Erigone as recently as March 20, 2014. Focusing upon the seven candidate stars in this paper, and taking Alpha Centauri B as an example which has a physical diameter some 86% of that of our Sun. At a distance of 4.36 light-years, its disc subtends an apparent angular diameter of just 0.00599" (5.99 milliarcseconds) on the sky. Given that the majority of objects in the asteroid main belt have orbital semi-major axes averaging around 2.6 AUs, their mean opposition distance from Earth averages at 1.6 AUs. By simple trigonometry, we note that it would take a small asteroid with a diameter of in the region of 7 kilometres in the main belt to completely occult the microscopic sized apparent disc of the star Alpha Centauri B, to hopefully enable one to "see" (and image) any of the exoplanets that might be in orbit around it. The question then arises as to how many asteroids in the main belt are (a) greater than 7 km in diameter, and (b) will in fact be able to wander significantly far from the ecliptic line in the sky to actually occult Alpha Centauri? Now Alpha Centauri is located on the geocentric celestial sphere 42.6 degrees south of the ecliptic plane of the Solar System. From the principles of celestial mechanics it follows that for a Solar System body to ever occult Alpha Centauri from the perspective of a geocentric observer, it must be in an orbit around the Sun with an inclination of at least 30 degrees or so relative to the plane of the ecliptic. Ephemerides simulations using the JPL Small Body database [4]show that 2791 Paradise, for example, which is a main belt asteroid of diameter 8.4km,orbital inclination 31.08 degrees will pass closer than a mere 20 arc minutes from Alpha Centauri on the evening of April 5th, 2241 (Figure 2). Outright occultations of Alpha Centauri by objects of similar size and orbit parameters as 2791 Paradise within much nearer time frames than 2241 are certainly possible.

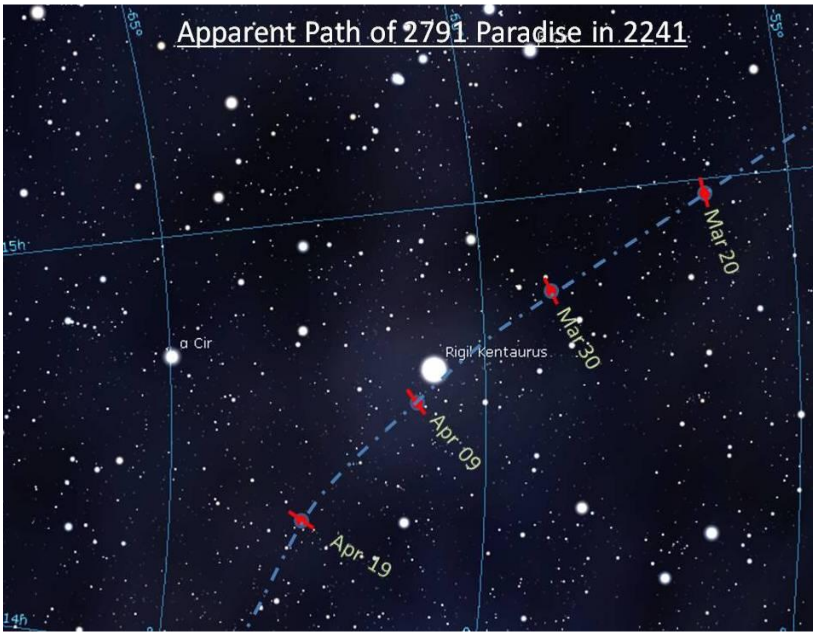


Figure2. *Asteroid number 2791 Paradise is set to pass Alpha Centauri very closely in 2241*

Going down to 7 km diameter objects in the asteroid main belt, there are in fact thousands with high inclination orbits that can wander as much as up to 30 or 40 degrees from the ecliptic line in the sky to occult all of the bright, nearby candidate stars highlighted in this paper.

Some astrophysical characteristics of these stars are shown in Table 3.

Table3. *Habitable Zones Around Candidate Stars*

	Distance (ly)	Physical Diam. (Sol*)	Apparent Diameter (mas)	App. Mag m_v	Abs. Mag M_v	Bolometric Correction[5]	Habitable Zone (AU)**	Habitable Zone (arc secs)
α Cen A	4.36	1.23	8.56	-0.01	4.38	-0.12	1.25	0.94
α Cen B	4.36	0.86	5.99	1.33	5.71	-0.30	0.73	0.55
Sirius A	8.60	1.71	6.03	-1.47	1.42	-0.15	5.02	1.90
Procyon A	11.46	2.05	5.43	0.34	2.66	-0.05	2.71	0.77
Altair	16.73	1.80	3.27	0.77	2.21	-0.05	3.33	0.65
Vega	25.04	2.60	3.15	0.03	0.58	-0.15	7.39	0.96
Fomalhaut	25.13	1.84	2.22	1.16	1.72	-0.10	4.27	0.55

*Units of Solar diameter

**HZ is defined as the distance from which the parent star would appear to shine at an apparent visual magnitude of -26.8 – i.e. the same as how bright the Sun appears on Earth. This has been derived using the distance modulus on the absolute magnitude (M_v) + the Bolometric Correction per Kaler (1997).

7. CONCLUSIONS

Detecting potentially life-bearing extrasolar planets orbiting within habitable zones are currently the targets of major ground and spaced-based efforts alike, such as the NASA Kepler mission to mention one example out of many. Using occultation methods will one day soon be part of our standard palette of techniques for both groundbased and space-based astronomy to discover and characterize exoplanets orbiting nearby stars. From the results in this paper, it is clear that ground based observers utilising large aperture telescopes could hope to detect the brightest of such planets during asteroidal occultations of the brightest stars within some 30 light-years from the Sun.

ACKNOWLEDGMENTS

I am especially grateful and express appreciation to the referees, namely Dr William Hartkopf at the USNO, Martin Beech and Dave Oesper for their helpful comments as a result of which the readability of this work has been much improved.

REFERENCES

[1] Ahad, A. ‘Magnitude model for extrasolar planets in the habitable zone’http://www.spacebanter.com/showthread.php?t=43598 (2004)

[2] NASA Planetary Factsheets:http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

[3] Wiegert, P. A. & Holman, M. J., ‘The Stability of Planets in the Alpha Centauri System’, Astronomical Journal,**113**, p. 1445-1450 (1997)

[4] JPL Small Bodies database online

[5] Kaler, James B. The Stars and their Spectra: Introduction to the Spectral Sequence. Cambridge University Press, p. 263 (1997)

Citation: *Abdul Ahad (2018). Detecting Habitable Exoplanets During Asteroidal Occultations International Journal of Scientific and Innovative Mathematical Research (IJSIMR), 6(9), pp.25-30. http://dx.doi.org/10. 20431/2347-3142.0609004*